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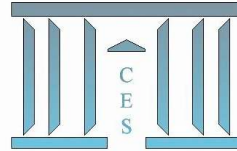
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## A theoretical framework for trading experiments

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**A general framework is suggested to describe human decision making in a certain class of experiments performed in a trading laboratory. We are in particular interested in discerning between two different moods, or states of the investors, corresponding to investors using fundamental investment strategies, technical analysis investment strategies respectively. Our framework accounts for two opposite situations already encountered in experimental setups: i) the rational expectations case, and ii) the case of pure speculation. We consider new experimental conditions which allow both elements to be present in the decision making process of the traders, thereby creating a dilemma in terms of investment strategy. Our theoretical framework allows us to predict the outcome of this type of trading experiments, depending on such variables as the number of people trading, the liquidity of the market, the amount of information used in technical analysis strategies, as well as the dividends attributed to an asset. We find that it is possible to give a qualitative prediction of trading behavior depending on a ratio that quantifies the fluctuations in the model.**

**Keywords:** decision making, game theory, complex systems theory, technical analysis, rational expectations.

### 1. INTRODUCTION

In order to gain new insight on how investors perceive investment possibilities as well as risks in financial markets, it appears important to confirm not only the background of theoretical studies on human decision making, but also to get knowledge from controlled experiments, where one can probe in detail the different assumptions of investment behavior. It is only recently that experimental Finance has begun to appear as a well-established field, the interest in particular sparked by the recognition in terms of the attribution of the Nobel Prize in Economics to Vernon Smith in 2002. However, so far the major part of experimental work in Finance has assumed (Vernon Smith included) human rationality and the ability of markets to find the proper price close to an equilibrium setting. Contrary to this approach Behavioural Finance takes a more practitioner-minded description of how actual decision making takes place in financial markets. It would therefore seem like a very natural approach to bridge the insight gained from Behavioral Finance and apply it to experiments done on financial markets. Interestingly, not much effort has been done in this direction. The main reason is maybe because the major part of research done in Behavioural Finance is concerned with how *individual* decision making takes place (Prospect Theory included (Tversky and Kahneman (1974), Kahneman and Tversky (1979), and Tversky and Kahneman (1991) )) and in a *static* setting, whereas price setting in financial markets is clearly an *aggregate* and *dynamic* phenomenon.

The efforts in this paper should be seen from such a viewpoint. Our theoretical foundation is based on a complex systems approach that places emphasis on social learning and group behaviour in order to understand the price formation in financial markets. The idea is that financial market participants are connected through their impact on the price as well as through the percolation of information through the group of market participants. For instance, “shocks” created by a large liquidation of a given market participant can have future impacts on the decision making of other market participants who in turn follow a similar decision to liquidate their positions. In the context where both dynamic behaviour as well as social learning and group behaviour are relevant, tools from complexity theory are particularly appealing, including for example agent-based modeling and game theory as presented here.

We are especially interested in discerning between two different moods (or states) of the population of investors, corresponding to i) investors using fundamental investment strategies as in the case of rational expectations and ii) the emergence of a speculative bias as seen in certain cases when investors use technical analysis strategies. The rational expectations case i) has been studied extensively in a large number of experiments under various situations and with different constraints (Smith (1962), Smith (1965), Plott and Smith (1978), Coppinger et al. (1980), Hommes et al. (2008)). In the simplest setup, which is included in our theoretical description further below, people trade shares of a given company based on their expectations of future dividends of the company. Throughout the experiment such expectations change due to the arrival of new information. The experiment ends with the closure of the company and the payout of the dividends to the participants in the experiments. It should be noted that in this case there is no incentive for the participants to speculate on the price itself since the full price of the company reflects the expected dividends payout *at the end of the experiment*. A case study was done for the opposite situation where expectations about dividends do *not* play any role, and reported in Roszczynska-Kurasinska et al. (2012). In this experiment only the price was available for the investment decisions of the group of participants. However as was shown in Roszczynska-Kurasinska et al. (2012), it requires coordination among the participants to profit from a speculation bias in this kind of experiments.

It should be noted that our approach differs from most of the Behavioral Finance/bounded-rationality literature in that the phenomena we study can only be understood by looking at the system level. In other words, although the phenomena that emerge depend on microscopic features of the agents, it is important to not only look at individual characteristics but to study the system as a whole. The state of the system (speculative or fundamentalist) is the macroscopic result of many microscopic decisions. We shall refer to this collective “choice” of the state of the system as “aggregate decision making”.

Our setup is conducive to answering a number of well-defined questions—that of how prices form, for instance—that the Rational Expectations Hypothesis (REH), or even the bounded REH cannot address. Once one mixes in the two ingredients that are the asset price and the dividends, the situation becomes ambiguous: REH suggests agents should make their trading decisions based on dividends, but the matter becomes far from trivial once the price of an asset and an end date are factored in. Are decisions based on the price of the asset in anticipation of future price behavior (speculative state), or are dividends the only drivers of agents’ trading decisions (fundamentalist state)?

In the following we introduce theoretical foundations encompassing the occurrence of both the speculative and the fundamentalist state. We do so by considering the “Dollar Game” (or “\$-Game”), which is an investment game that combines the two key ingredients that are dividends and the asset price (Vitting Andersen, Sornette (2003)). Although simple in principle, the \$-Game yields rich system dynamics, the complexity of which can be acted upon by the choice of system parameters (memory length, liquidity, etc.). As will be seen, this thereby creates a dilemma in terms of the investment strategies of the participants. The pure cases i) and ii) will appear as special cases of the general theory.

This feature of the \$-Game lends itself well to study using a general well-known theory of phase-transitions found in Physics: the Ginzburg-Landau theory (henceforth referred to as “the GL theory”) which we describe later.

## 2. THE \$-GAME

The \$-Game was inspired by the Minority Game (MG) introduced in 1997 by Ye-Cheng Zhang and Damien Challet (Challet and Zhang (1997), Challet and Zhang (1998) as an agent-based model proposed to study market price dynamics (Zhang (1998), Johnson et al. (1999), Lamper et al. (2002)). The MG was introduced following a leading principle in Physics, that in order to solve a complex problem one should first identify essential factors

history, $\vec{h}(t)$	action, $a_i^j(t)$
000	1
001	-1
010	-1
011	1
100	-1
101	-1
110	1
111	-1

Example of a strategy

at the expense of trying to describe all aspects in detail. Similar to the Minority Game, the \$-Game should be considered as a “minimal” model of a financial market.

Formally,  $N$  players (or agents) simultaneously take part in a one-asset financial market over a horizon of  $T$  periods. At each period,  $t \leq T$ , each player  $i$  chooses an action  $a_i(t) \in \{-1, 1\}$ , where action “1” is interpreted as “buy” and action “-1” as “sell”. Players are assumed to be boundedly rational, in the sense of using only a limited information set upon which to base decisions. In the version of the \$-Game presented in this paper, the agents use two different types of investment strategies, technical analysis strategies and fundamental analysis strategies. Concerning the decision making related to technical analysis, each player observes the history of past price movements, which is limited to the size of their memory,  $m \in \mathbb{N}$ . Each player has at his/her disposal a fixed number of  $s$  strategies which are randomly assigned at the beginning of the game. It follows that player  $i$ ’s,  $j$ ’th strategy,  $a_i^j$ , is a mapping from the set of histories of size  $m$  to  $\{-1, 1\}$ . We denote by  $\vec{h}(t) \in \{0, 1\}^m$  the history vector that agents observe in period  $t$  before taking the action of either buying or selling an asset. We interpret “1” to represent an up move of the market (an increase in the asset price) and “0” corresponds to a down move of the market (a fall in the price of the asset). These assumptions are equivalent to having agents behave as technical analysts who use lookup tables to determine their next move. Table 1 shows an example of a strategy for  $m = 3$ :

A strategy therefore tells an agent what to do given the past market behavior. If the market went down over the last three days, the strategy represented in Table 1 suggests that now is a good moment to buy (000  $\rightarrow$  1) in Table 1. If instead the market went down over the last two days and then up today, the same strategy suggests that now is a good moment to sell (001  $\rightarrow$  -1) in Table 1. While a single strategy recommends an action for all possible histories (of length  $m$ ), we also allow for agents to adopt different strategies over time. Namely, agents keep a record of the overall payoff each strategy would have yielded over the entire market history (i.e. not limited to  $m$  periods prior) and use this record to update which strategy is the most profitable. In every time period agent  $i$  therefore choses the **best** strategy (in terms of payoff, see definition below) out of the  $s$  available. This renders the game highly non-linear: as the price behavior of the market changes, the best strategy of a given agent changes, which then can lead to new changes in the price dynamics. The action of the best strategy of agent  $i$  at time  $t$ , is denoted by  $a_i^*(t)$ . We denote by  $(a^*(t))_i \in \{-1, 1\}^{N \times T}$  the *action profile* of the population, where  $\vec{a}^*(t) = (a_1^*(t), \dots, a_N^*(t)) \in \{-1, 1\}^N$  corresponds to the action played by the  $N$  agents in period  $t$ .

The payoff  $\pi$  of the  $i$ th agent’s  $j$ th strategy,  $a_i^j$ , in period  $t$  is determined as follows:

$$\pi[a_i^j] = a_i^j(t-1) \sum_{k=1}^N a_k^*(t) \quad (1)$$

The return  $r(t)$  of the market between period  $t$  and  $t+1$  is assumed to be proportional to the order imbalance  $\sum_{k=1}^N a_k^*(t)$ :

$$r(t) = \sum_{k=1}^N a_k^*(t) / \lambda \quad (2)$$

with  $\lambda$  a parameter describing the liquidity of the market. Therefore the payoff of a given strategy (1) can be expressed in terms of the return of the market in the next time period as:

$$\pi[a_i^j] = a_i^j(t-1) \lambda r(t) \quad (3)$$

From (1) one can see that the payoff depends on two different times: the individual decision at time  $t - 1$  and the aggregate "decision" at time  $t$ . Such a feature of the payoff function is illustrative of real financial markets, where traders decide to enter a position in a market at time  $t - 1$ , but do not know their return until the market closes the next day (time  $t$ ). This is especially clear from (3) where it can be seen that the  $\$$ -Game rewards a given strategy that at time  $t - 1$  predicted the proper direction of the return of the market  $r(t)$  in the *next* time step  $t$ . The larger the move of the market, the larger the gain/loss depending on whether the strategy properly/improperly predicted the market move. Therefore in the  $\$$ -Game, agents correspond to speculators trying to profit from predicting the direction of price change.

In addition to technical analysis strategies that try to profit from price changes, we also consider strategies that try to profit from information of the fundamental value of an asset  $P_f(t)$ .  $P_f(t)$  is determined entirely from future expectations about the dividends  $d(t)$  attributed to the asset at the end of the experiment. Whenever  $P(t) > P_f(t)$  a fundamental strategy therefore gives the recommendation to sell, whereas if  $P(t) < P_f(t)$  it recommends buying. Furthermore in order to take into account a diminishing use of such strategies in a purely speculative phase when the price  $P(t) \gg P_f(t)$ , the probability to use a fundamental strategy is taken from a Poisson distribution  $\gamma \exp(-\gamma)$  with  $\gamma = \frac{P(t) - P_f}{d}$ .

To sum up, the  $\$$ -Game as described in this article can be described in terms of just five parameters:

- $N$  - The number of agents (market participants).
- $m$  - The memory length used by the agents.
- $s$  - The number of strategies held by the agents. It should be noted that the  $s$  strategies of each agent is chosen randomly (corresponding to a random column of '0's and 1's in table 1) in the total pool of  $2^m$  strategies at the beginning of the game.
- $\lambda$  - The liquidity parameter of the market.
- $d(t)$  - The future expectations about the dividends paid at the end of the experiment. To simplify,  $d(t)$  will be taken constant in time  $t$  in this paper.

The dynamics of the  $\$$ -Game are driven by nonlinear feedback because each agent uses his/her *best* strategy at every time step. As the market changes, the best strategies of the agents change, and as the strategies of the agents change, they thereby change the market. Formally one can understand such dynamics by representing the price history  $h(t) = \sum_{j=1}^m b(t-j+1)2^{j-1}$  as a scalar where  $b(t)$  is the bit representing the direction of price movement at time  $t$  (see table 1). The dynamics of the  $\$$ -Game can then be expressed in terms of an equation that describes the dynamics of  $b(t)$  as:

$$b(t+1) = \Theta\left(\sum_{i=1}^N a_i^*(h(t))\right), \quad (4)$$

with  $\Theta$  a Heaviside function. The nonlinearity of the game can be formally seen from:

$$a_i^*(h(t)) = a_i^{\{j | \max_{j=1, \dots, s} \{\Pi[a_i^j(h(t))]\}}(h(t)), \quad \Pi[a_i^j(h(t))] = \sum_{k=1}^t a_i^j(h(k-1)) \sum_{i=1}^N a_i^*(h(k)) \quad (5)$$

Inserting the expressions (5) in expression (4) one obtains an expression that describes the  $\$$ -Game in terms of just one single equation for  $b(t)$  depending on the values of the variables ( $m, s, N, \lambda, d$ ) and the random variables  $a_i^j$  (i.e. their initial random assignments). A major complication in the study of this equation happens because of the non-linearity in the selection of the best strategy. For  $s = 2$  however the expressions simplifies because one only need to know the relative payoff  $q_i \equiv \pi[a_i^1] - \pi[a_i^2]$  between two strategies (Challet, Marsili (1999), Challet, Marsili (2001)). For this special case it was shown in Roszczynska-Kurasinska et al. (2012) that the Nash equilibrium for the  $\$$ -Game with only technical analysis strategies (with no cash nor asset constraints) is akin to that of Keynes' "Beauty Contest" where it becomes profitable for the subjects to guess the actions of the other participants. The optimal state is then one for which all subjects cooperate and take the *same* decision (either buy/sell).

### 3. GINZBURG-LANDAU THEORY

To describe further the competition between technical analysis trading strategies and fundamental analysis trading strategies as used in the \$-Game, we suggest to borrow a description from Physics where different states of a system can be characterized via a so-called free energy  $F$ .  $F$  in that case plays a central role, since its minimum determines how the state of the system will appear.  $F$  can be written as  $F = E - TS$  with  $E$  the energy of the system,  $T$  the temperature and  $S$  the entropy which one can think of as representing how much disorder there is in a given system. From the definition of  $F$  we can see that the state of a system is determined by a struggle between two different forces, one representing “order”, this is the  $E$  term, and the other term representing “disorder” given by the  $TS$  term. We suggest a similar struggle of “forces” to be present in the trading experiments.

The competition between order and disorder as described by  $F$ , can be understood in more detail by considering the example given by the Ising model, which is a model of ferromagnetism. For the Ising model the energy  $E = -J \sum_{\langle i,j \rangle} s_i s_j$  with  $s_i, s_j$  representing the atomic “spins” of a material. The  $\langle \rangle$ -notation in the summation indicates that the sum is to be taken over all nearest neighbors pair of spins. Each spin itself can be thought of as a mini magnet. In the two-dimensional Ising model the spin  $s_i = 1$  if the spin is “up” and  $s_i = -1$  if the spin is “down”. Taking the coupling strength between spins  $J$  positive, the minimum energy  $E_{\min}$  of the system is simply given by either all spins up ( $s_i \equiv 1$ ), or all spins down ( $s_i \equiv -1$ ). For temperature  $T = 0$  the minimum of the energy  $E$  is therefore also the minimum of the free energy  $F$ . However as soon as  $T > 0$ , the finite temperature will introduce fluctuations of the spins introducing thereby a non-zero contribution to the entropy  $S$ . The larger the temperature  $T$  the larger this tendency, until at a certain temperature  $T_c$  above which order has completely disappeared - the system is in a disordered state. Order in the case of the Ising model is measured by the magnetism, which is just the averaged value of the spin  $m = E(s_i)$ .

GL theory introduces the idea that we can in general understand such order-disorder transitions mentioned above by expanding the free energy in terms of the order parameter  $m$ . Specifically write:

$$F = C + am + \alpha m^2 + bm^3 + \beta/2m^4 + \dots \quad (6)$$

Using now the symmetry argument that there should be no difference in the free energy between the two states with respectively either all  $s_i \equiv 1$  or  $s_i \equiv -1$ , all odd order terms in  $m$  disappear in (6). Taking furthermore the derivative (in order to find its extreme) we end up with the equation at a minimum of  $F$ :

$$0 = m(\alpha + \beta m^2) \quad (7)$$

(7) has the trivial solution of the magnetization  $m = 0$ , this is the high temperature solution and describe the disordered state. Taking  $\beta$  positive, the other non-trivial solution happens for negative  $\alpha$ ,  $m^2 = -\alpha/\beta$ . By writing  $\alpha = (T - T_c)$  one sees that the magnetization scales as  $(T - T_c)^{1/2}$  for temperatures below  $T_c$ . The exponent of 0.5 is the so called “mean field” or GL exponent of the transition.

We now propose to consider in similar terms the competition in the trading experiments between profit from speculation obtained through trend-following, versus the tendency to destroy such trends due to mean reversion towards the fundamental price (Vitting Andersen (2010)). The general tendency to create either a positive/negative price trend corresponds to “order” whereas either the lack of consensus or the mean reversion to the fundamental price value will destroy such order. To make the analogy with our discussion above, we introduce what one could call the “free profit” given by two terms  $F_P = P - TS$ .  $P$  is the profit of the ordered state which for  $T = 0$  corresponds to a continuous up/down trend of the market.  $S$  is an entropy term that destroys the ordered state, and  $T$  is the “temperature” which will be introduced below.

As discussed beforehand, the payoff of a strategy in the \$-Game describes the profit for the given strategy. Agents in a Nash equilibrium are characterized by using the same strategy over time, therefore such a strategy has to be optimal. We can then write the total profit  $P$  for the system of traders in a Nash equilibrium of the \$-Game as:

$$P(t) = \sum_i \pi[a_i^*] \quad (8)$$

$$= \sum_{i=1}^N \sum_{j=1}^N a_i^*(t-1) a_j^*(t) \quad (9)$$

We note the resemblance of (9) to the Ising model described above. One major difference with respect to the Ising model however is the “interaction” between traders, since (9) says that trader  $j$ ’s action at time  $t$  has an impact on trader  $i$ ’s profit from the action he/she took at time  $t - 1$ . Therefore the “interaction” is seen to be “long-ranged” in (9) whereas the interaction is local (it only concerns nearest-neighbors) for the Ising model.

Similar to (6) we can introduce an order parameter and expand the “free profit” in terms of this parameter. In the case of the Ising model the order parameter is given by the spatial average of the local order (the magnetization). In the trading setup we suggest to consider the local order  $o$  expressed in terms of the order imbalance:  $o = 1/N \sum_{i=1}^N a_i^*$  which from (2) is seen to be proportional to the return. The parameter  $o$  varies between -1 (all agents decide to sell) and 1 (all agents decide to buy). In the case where one can neglect the dividends (the experiment described in Roszczynska-Kurasinska et al. (2012))  $o \rightarrow \pm 1$  so this corresponds to the complete ordered outcome. For the experiments performed under the assumptions of rational expectations the price converges to the fundamental price in the end of the experiments and we get  $o \rightarrow 0$  for  $t \rightarrow t_n$  with  $t_n$  the duration of the experiment.

Applying now the GL idea and expanding  $F_p$  in terms of  $o$ , one ends up with the very same conditions (7) to determine  $o$ , except that the extreme (extremes) now describes a maximum (maxima) instead of a minimum (minima) as was the case for  $F$ . Note that all odd order terms of  $o$  disappear since there is no difference in the profit that traders obtain in shorting the market compared to going long. Figure 1 illustrates the expansion of  $F_p$  ( $y$ -axis) as a function of  $o$  ( $x$ -axis) for the two cases: i) the  $T > T_c$  solution (i.e. the disordered state corresponding to no trend in the experiments  $o = 0$ ) can be seen as the maximum of the solid line, whereas the two  $T < T_c$  solutions (i.e. the ordered state corresponding to a certain trend in the experiments  $o \neq 0$ ) can be found as the maxima of the dashed line.

One of the main implications of the GL theory is the existence of a nontrivial transition from a high “temperature” disordered state in the trading experiments where traders don’t create a trend over time, to a low “temperature” state characterized by trend following. A “temperature” can now be defined via the randomness of the model as will be explained in the following. Randomness enters the \$-Game through the initial conditions in the assignments of the  $s$  strategies to the  $N$  traders in the game. In order to create a given strategy one has to assign randomly either a 0 or a 1 for each of the  $2^m$  different price histories. Therefore the total pool of strategies increases as  $2^{2^m}$  versus  $m$ . However many of these strategies are closely related - take e.g. table 1 and change just one of the 0’s to a 1, this thereby creates a strategy which is highly correlated to the one seen in table 1. In Challet, Zhang (1997), Challet, Zhang (1998), it was shown how to construct a small subset of size  $2^m$  of independent strategies out of the total pool of  $2^{2^m}$  strategies. As suggested in (Savit et al. (1999) and Challet, Marsili (2000)), a qualitative understanding of the MG can then be obtained by considering the parameter  $\alpha \equiv \frac{2^m}{N}$ . However as pointed out in Zhang (1998) the ratio  $\alpha' = \frac{2^m}{N \times s}$  seems intuitively to be more relevant since this quantity describes ratio of the total number of relevant strategies to the total number of strategies held by the traders. Taking into account the presence of a fundamental value strategy we therefore introduce the ratio  $T = \frac{2^m + 1}{N \times s}$  to describe the temperature in the simulations of the \$-Game presented in the following. The relation of  $T$  to the fluctuations of the system becomes clear when one consider that when sampling the variance of a small sample is larger than the variance of a large sample (a fact called “the law of *small* numbers” in Psychology/Behavioral Finance (Tversky, Kahneman (1974), Kahneman, Tversky (1979), Tversky, Kahneman (1991))). Therefore when the sample of strategies held by the  $N$  traders is small with respect to the total pool of relevant strategies, this corresponds to the large fluctuations, large temperature case. Vice versa a large sample of strategies held by the  $N$  traders therefore corresponds to a small temperature case as seen from the definition of  $T$ .

#### 4. RESULTS

Figure 2 shows three different results representing typical market behavior corresponding to fundamental price behavior, as well as speculative behavior in an increasing/decreasing market. Figure 3-4 show histograms representing respectively speculative behavior (blue) or fundamentalist behavior (red) as outcomes in a setup of the \$-Game corresponding to a trading experiment with given realizations of the 5 parameters ( $N, m, s, \lambda, d$ ). The histograms in figure 2 represents simulations performed with  $s = 2$  whereas the histograms in figure 3 were done for simulations with  $s = 18$ . The different histograms were obtained from an ensemble average of 200 simulations of the \$-Game where each realization of the game were run for up to  $200 \times 2^m$  time steps. A speculative state was determined whenever  $m$  successive price changes had occurred whereas a fundamental state was characterized by



the price fluctuation within a 50 percentage range of the fundamental value  $P_f$ . White in the figures represents the cases where neither a definite speculative nor fundamental state could be defined.

We first notice the somewhat surprising fact that the dividends  $d$  as well as the liquidity of the market  $\lambda$ , only seem to have a quite limited impact on the final state of the market. In particular for the smallest  $m$  values ( $m = 3, 5$ ) increasing dividends appear to have a somewhat stabilizing effect allowing for slightly more fundamental value states. The same stabilizing trend appears to be at play as one increases the liquidity of the market, but again, this tendency appear to be very weak. A much clearer tendency is seen with respect to increasing speculation when increasing the number of traders  $N$ , respectively decreasing the amount of information  $m$  used in the decision making of the technical analysis trading strategies. A larger number of strategies  $s$  assigned to the traders is also seen to enhance speculation (compare figure 3 and figure 4).

One of our main results is that a qualitative behavior of a trading experiment can be predicted depending on the given value of  $T$ . In particular with respect to expectations about the outcome in an experimental setup of the market model, such an understanding is important (Bouchard et al. 2012). The fact that  $T$  determines the outcome of trading behavior can be seen by changing the nominator and denominator by the same factor, which then should lead to invariant behavior in terms of trading decisions. This means that for example the ( $m = 3, N = 11$ ) (i.e.  $T = 0.72/s$ ) trading behavior for a given  $\lambda$  and  $s$  should fall in between the ( $m = 5, N = 101$ ) (i.e.  $T = 0.32/s$ ) and ( $m = 8, N = 101$ ) (i.e.  $T = 1.27/s$ ) cases. From figure 2 and figure 3 this is seen indeed to be the case. Similarly comparing figure 3-4 it is seen that increasing (/decreasing)  $N$  and decreasing (/increasing)  $s$  by the same amount leads to two systems behaving similarly in terms of investment profile (compare  $N = 101$  rows in figure 3 to  $N = 11$  rows in figure 4). These results underscore the importance of the parameter  $T$  when it comes to the understanding of the aggregate decision making in the model.

## 5. CONCLUSION

A general framework has been suggested to describe the human decision making in a certain class of experiments performed in a trading laboratory. Our framework allows us to predict the outcome of such type of trading experiments in terms of when to expect a fundamental versus a speculative state. We have shown how a qualitative understanding could be found depending on just one parameter, representing the fluctuations of the model. Our findings give certain guidance with respect to the implementation of trading experiments performed in a trading laboratory (Bouchard et al. (2012)).

## 6. ACKNOWLEDGMENTS

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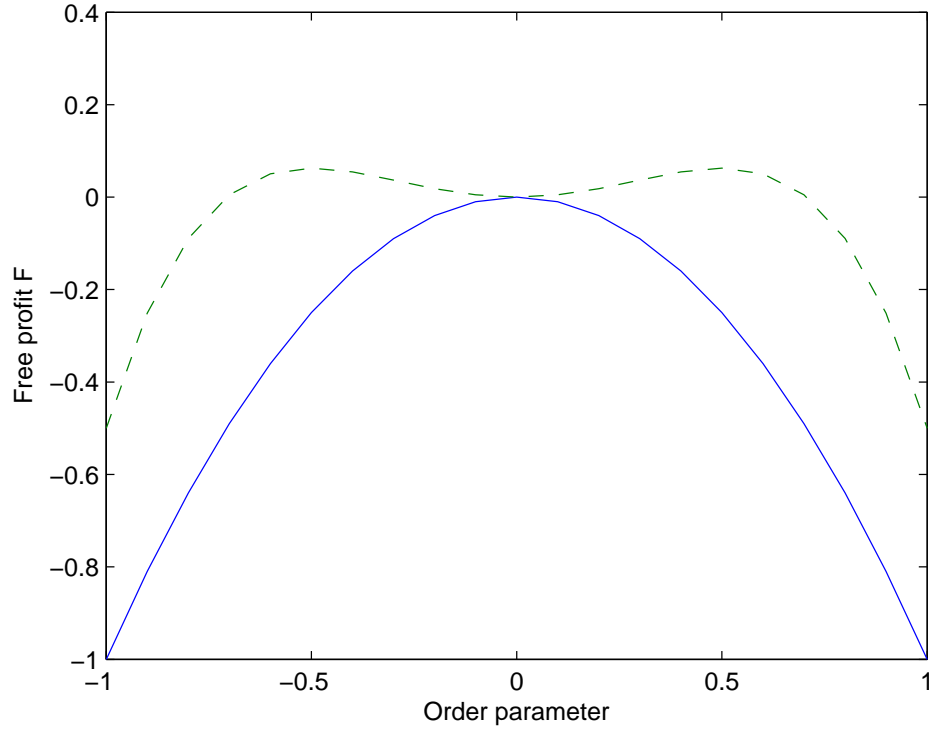


FIG. 1: Illustration of the “Free Profit”  $F_p$  as a function of the order parameter  $o$  for two different “temperatures” corresponding to  $T > T_c$  (solid blue line), and  $T < T_c$  (dashed green line) respectively.

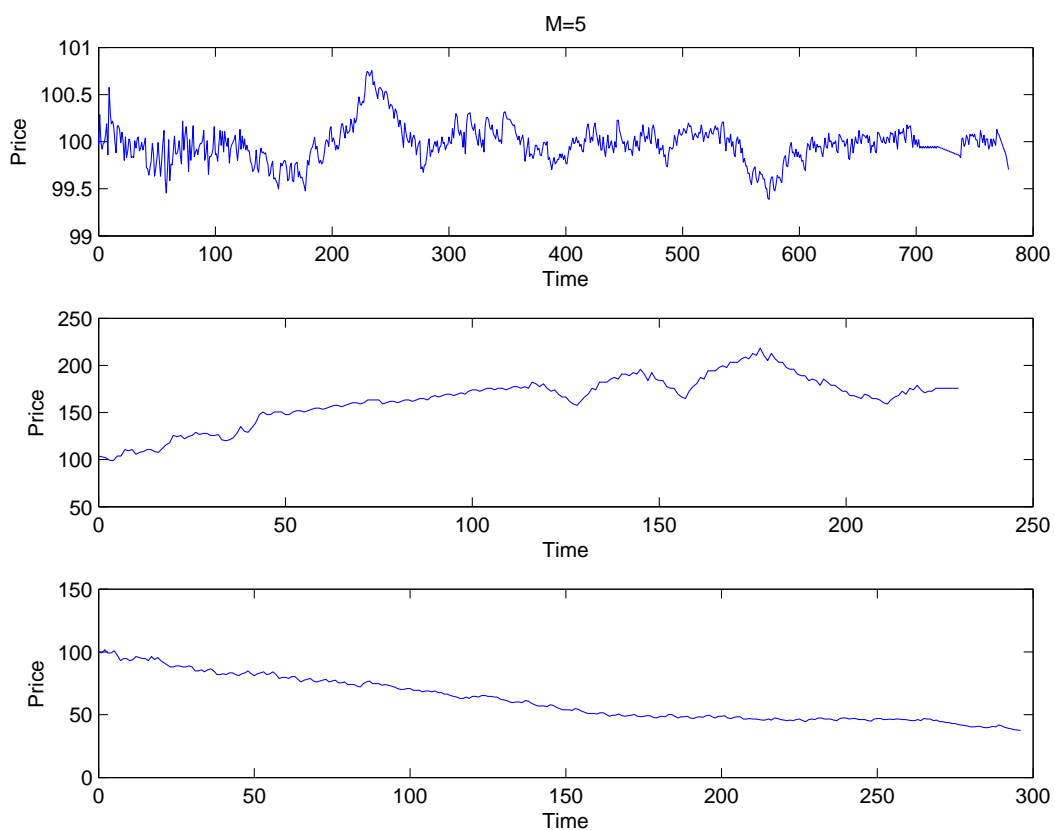


FIG. 2: 3 different examples corresponding to speculative price behavior in a fundamental value/increasing/decreasing market.

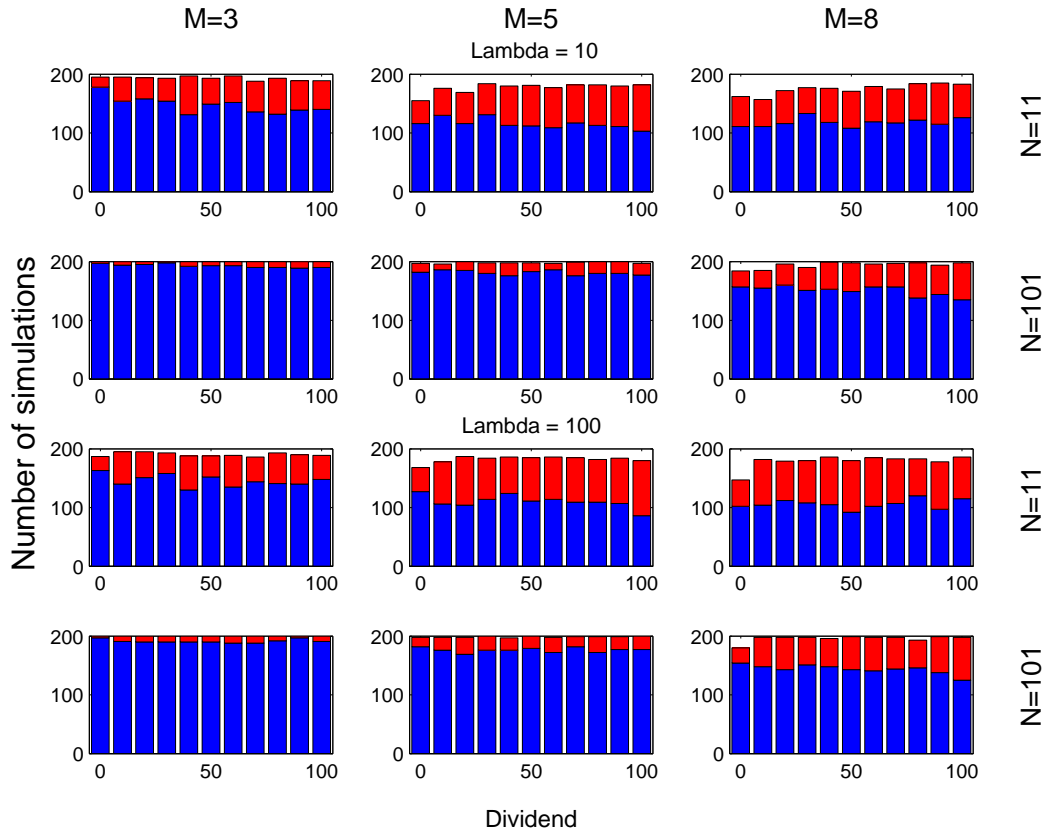


FIG. 3: Histograms representing respectively speculative behavior (blue) or fundamentalst behavior (red) as outcomes in a setup of the  $\$$ -Game for  $s = 2$  with given parameter values of  $(N, m, \lambda, d)$ .

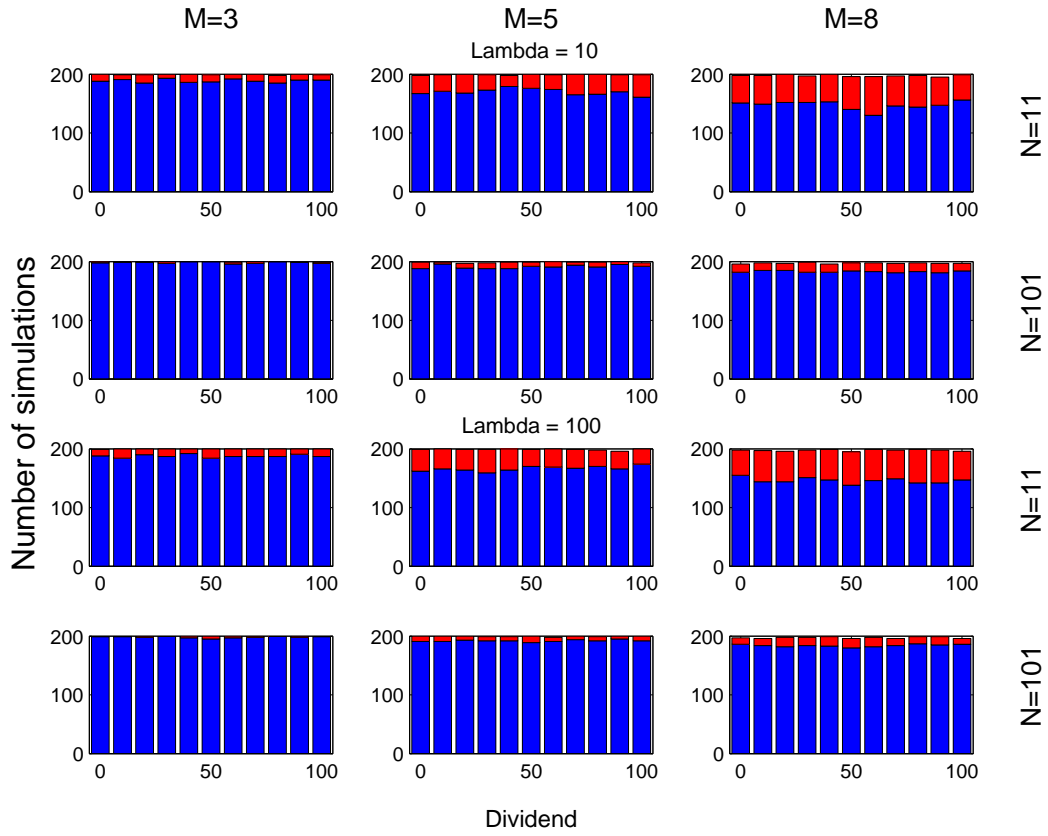


FIG. 4: Histograms representing respectively speculative behavior (blue) or fundamentalst behavior (red) as outcomes in a setup of the  $\$$ -Game for  $s = 18$  with given parameter values of  $(N, m, \lambda, d)$ .